

Lattices and Barrier Options Computational Notes

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Often in working with lattices, we need to adjust some nodes of the lattice for the stock price or the lattice for the option price. For example, the nodes for the stock prices sometimes must reflect dividends. For the option lattice, the value at a node might not be merely a function of the next period's option value and the associated probabilities. For example, in constructing the lattice for an American call we need to reflect the opportunity for immediate exercise at each node. For a barrier option, we need to consider the effect of the barrier on the option price at each node.

To make this more concrete, let us consider an up-and-out barrier call option with the following particulars:

$S = \$80$
 $E = \$70$
 $r = 0.07$
 $\sigma = 0.3$
 $T - t = 120$ days
Knock-out Price = \$100

These values imply:

$U = 1.1044$
 $D = 0.9055$
 $\Pi_U = 0.5138$
 $\Pi_D = 0.4962$

The next page shows the lattices for the stock and the option. Notice that the highest terminal stock price in the lattice is \$107.77. A plain vanilla call with an exercise price of \$70 would pay \$37.77 at this node. However, the value for the node at that point is zero for the knock-out option. This zero value reflects the fact that the option gets knocked out because the stock price exceeds the knock-out price of \$100. This example illustrates the basic principle for pricing a barrier option:

1. Construct the stock price lattice in the normal manner.
2. Begin working through the option lattice as one would for a plain vanilla option, except at each node, check against the barrier condition:
 - A. If the barrier condition is a knock-out condition, and the option is knocked-out, insert a zero value at that node in the option lattice. Otherwise, the correct option value at that node is the same as a corresponding plain vanilla option.
 - B. If the barrier condition is a knock-in condition, and the option is knocked-in, insert the value of the plain vanilla option at that node in the barrier option lattice. Otherwise the correct value at that node is zero, because the option is not knocked in at that node.

Periods	0	1	2	3
				107.77
			97.58	
Stock Lattice	80.00	88.35	80.00	88.35
		72.44		72.44
			65.59	
				59.39
				37.77
Plain Vanilla Call Lattice	12.78	19.42	28.11	18.35
		5.97	10.53	2.44
			1.24	0.00
				0.00
				0.00
Knock-Out Call Lattice	7.77	9.60	8.85	18.35
		5.97	10.53	2.44
			1.24	0.00
				0.00